

Alternating Series Error Bound

Given

$$\sum (-1)^n a_n,$$

the remainder/error is found by looking at the first unused term

$$|R_n(x)| = |f(x) - P_n(x)|$$

Because the series is alternating with individual terms that decrease in value to zero, the remainder is less than or equal to the absolute value of the first unused term.

$$|f(x) - P_n(x)| \leq |a_{n+1}|$$

Example 1:

Find the error involved in calculating the sum of the first six terms of the series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

Example 2:

Find the error involved in calculating the sum of the first six terms of the series:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

Example 3:

Estimate the error of $\sin(0.2)$ from the Taylor Polynomial of order 4.