Chapter 2 (Unit 1) Review

- IVT
- $\lim_{x\to 0}\frac{\sin x}{x}=1$
- Piecewise
- Continuity, Discontinuity
- Limits: numerically, graphically, & analytically
- Infinite limits (including HA and VA)
- Average Rate of Change, Instantaneous Rate of Change

1. a. Use your graphing calculator and the Intermediate Value Theorem to show that the function $f(x)=x^3+x^2-2x+3$ has a c in the interval [1,2] such that f(c)=5.

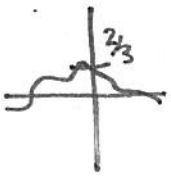
f cont b/c polynomial.
$f(1)=3$
$f(2)=11$

Since f is cont on [1,2] and $f(1)<5$ and $f(2)>5$, then $\exists$ a #, c, on [1,2] s.t. $f(c)=5$.

b. How could using IVT fail?

f discontinuous
or if f(a) & f(b) are both > or < k.

2. Use a graphing utility to find $\lim_{x\to 0}\left(\frac{\sin 2x}{3x}\right)$. Then, find the limit analytically.



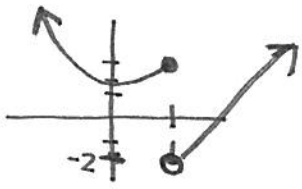
$\lim_{x\to 0}\frac{\sin 2x}{3x}=\lim_{x\to 0}\frac{2\cdot\sin 2x}{3\ 2x}$

$\lim_{x\to 0}\frac{2\cdot\sin 2x}{2\cdot 3x}=\frac{2}{3}\cdot 1$

$=2/3$

3. Sketch the graph of the following function. Using the definition of continuity, find the x-values for which f is not continuous.

$$f(x)=\begin{cases}x^2+2 & x\le 1\\ 3x-5 & x>1\end{cases}$$



$f(1)=1^2+2=3$

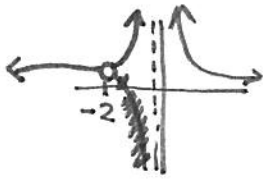
$\lim_{x\to 1^-}(x^2+2)=3$
$\lim_{x\to 1^+}(3x-5)=-2$
$\}\neq$, so $\lim_{x\to 1}f(x)$ DNE

f not cont @ x=1 b/c $\lim_{x\to 1^-}f(x)\neq\lim_{x\to 1^+}f(x)$ (JUMP)

4. Find the discontinuities for $f(x)$. Label them as removable or nonremovable. Sketch a graph of the function.

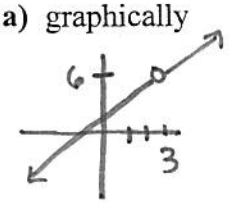
$f(x)=\frac{x+2}{x^3+2x^2}=\frac{x+2}{x^2(x+2)}=\frac{1}{x^2}$ → nonremovable @ x=0 (VERTICAL ASYMPTOTE)

↳ removable @ x+2=0, x=-2 (HOLE)



5. Find $\lim_{x\to 3}\dfrac{x^2-9}{x-3}$

a) graphically



6

b) numerically

x	2.9	2.99	2.999	3	3.001	3.01	3.1
f(x)	5.9	5.99	5.999		6.001	6.01	6.1

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c) analytically

$\lim_{x\to 3}\dfrac{(x+3)(x-3)}{x-3}$

$= \lim_{x\to 3}(x+3)$

$= 6$

6. At what values of x is $f(x)=\dfrac{x-2}{x^2-x-2}$ discontinuous?

$f(x)=\dfrac{x-2}{(x+1)(x-2)}$

$f(x)=\dfrac{1}{x+1}$

f discont @ $x=2$, $x=-1$ (HOLE) (ASYMPTOTE)

7. Find: **a)** $\lim_{x\to 2-}\dfrac{x-1}{x-2} = -\infty$

b) $\lim_{x\to 3+}\dfrac{5}{x-3} = \infty$

8. Find: **a)** $\lim_{x\to\pi}\tan x = 0$

b) $\lim_{x\to\pi}\sec x = -1$

c) $\lim_{x\to\frac{\pi}{2}}\csc x = 1$

show algebraically, graphically, (analytically) or numerically

9. Study your quiz, your homework, and your notes. Remember, you need to show work or write words to explain each problem for the test—even the multiple-choice questions. There are 7 non-calculator and 7 calculator problems on the test. When you finish the 7 non-calculator questions, you will turn in the non-calculator part and receive the calculator part. You must budget your time yourself. You will have the whole period to work on it.