

# Bounce Back: The Exponential Pattern of Rebound Heights

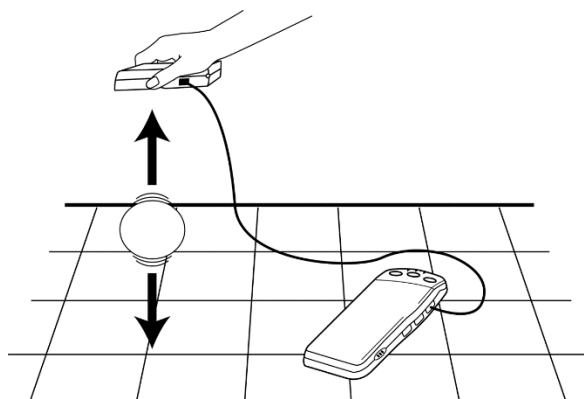
When a ball bounces up and down on a flat surface, the maximum height it reaches decreases from bounce to bounce. In fact, the maximum height decreases in a very predictable way for most types of balls. The relationship between the maximum height attained by the ball on a given bounce (which we will call the *rebound height*) and number of bounces that have occurred since the ball was released is an exponential

$$y = ab^x$$

where  $y$  represents the rebound height,  $x$  represents the bounce number,  $a$  is the release height, and  $b$  is a constant that depends on the physical characteristics of the ball used.

It's easy to see where this model comes from: Let's say that the ball is released from height  $a$ . Then on each bounce, it rebounds to a fraction  $b$  of the previous maximum height. After zero, one and two bounces the ball will attain a maximum height of  $a$ ,  $ab$ ,  $(ab)b = ab^2$ , and so forth. The relation above is generalized for any  $x$  number of bounces.

In this activity, you will collect motion data for a bouncing ball using a Motion Detector. You will then analyze these data to test the model  $y = ab^x$ .



## OBJECTIVES

- ⊕ Record the successive maximum heights for a bouncing ball.
- ⊕ Model the bounce height data with an exponential function.



## MATERIALS

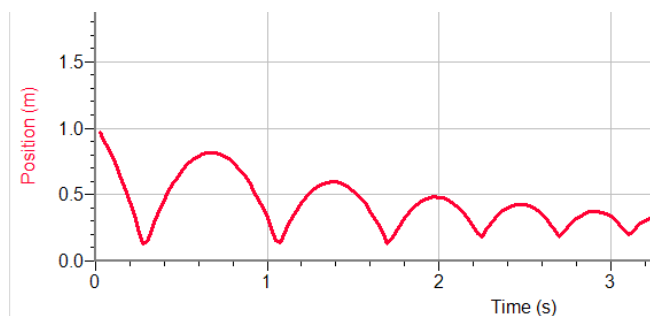
- computer (one per group needed)
- Vernier computer interface (grey device)
- Logger Pro (software on computer)
- Motion Detector (green device)
- ball (a basketball works well)

## GROUP ROLES

- Device Operator (**DO**)– connects & operates the motion detector
- Computer User (**CU**) – runs the Logger Pro software
- Bouncer (**B**)– releases ball
- Supervisor/Data Recorder (**S**)– ensures accurate data is collected & records results


## PROCEDURE

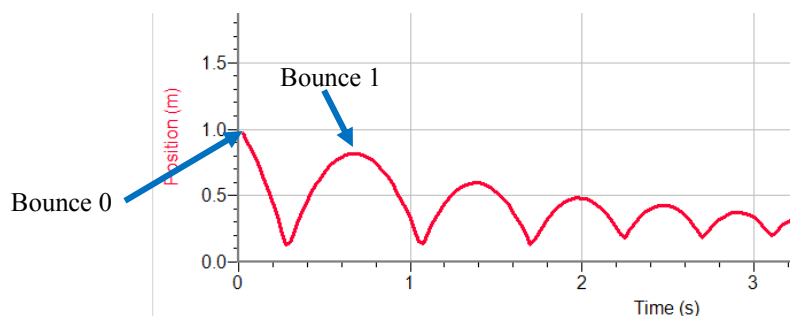
1. **(DO/S)** Connect the Motion Detector to the DIG/SONIC 1 channel of the Vernier computer interface.
2. **(CU/S)** Start *Logger Pro*.  
In the *Real-World Math with Vernier* folder, open the file “18 Bounce Back.”
3. **(DO/S)** Hold the Motion Detector about 1.5 m above the floor, pointing straight downward.  
**(B/S)** Practice dropping the ball so that it bounces straight up and down beneath the Motion Detector. Let the ball bounce on a smooth, level surface, and minimize the ball’s sideways travel. Dropping the ball from a height of about 1 meter works well. Do not allow anything to obstruct the path between the Motion Detector and the ball while data are being collected.  
**(DO/S)** If the ball does move sideways slightly, move the Motion Detector **horizontally** to stay above the ball.
4. **(DO/S)** To make the floor the zero reference, hold the Motion Detector as you practiced.  
**(CU/S)** Click the  button.
5. **(DO/S)** With the Motion Detector still at the same height at which you zeroed it,  
**(CU/S)** click  to begin data collection.  
**(B/S)** Drop the ball under the detector as you practiced earlier. Data collection will run for five seconds.
6. **(CU/S)** Examine the position *vs.* time graph. Your distance *vs.* time graph should contain a series of **at least five smoothly changing parabolic regions**. Ideally, your data should look similar to this:



- (S)** Check with your instructor if you are not sure whether you need to repeat the data collection. Redo the previous step if you need to repeat data collection.

## ANALYSIS

- To compare your position data to the model for bounce height, read the maximum height of each bounce from your distance vs. time data.
  - (CU/S) Click once on your graph to make it active.  
(CU/S) Click on the Examine button, , to read values from your graph.
  - (CU/S) Use the mouse to trace across your graph. Start with the initial release height, and call that bounce number zero.  
(S) Record the consecutive maximum rebound heights for the next five bounces. Record these values, rounded to the nearest 0.001 m, in your data table.



Bounce number	Maximum height (m)
0	
1	
2	
3	
4	
5	

- Now enter the information from your data table into lists on your computer. Your table has columns for Bounce Number, N, as well as the Maximum Height attained.

- (CU/S) Click in the first cell in the Bounce Number column.  
Enter Bounce #s 0 to 5.

Time (s)	Position (m)	Latest N	Max Height (m)
0.025	0.971		
0.050	0.911		
0.075	0.848		
0.100	0.779		
0.125	0.704		
0.150	0.624		

- (CU/S) Click in the first cell in the Maximum Height column, and enter the initial release height and the subsequent maximum heights for each bounce in turn.

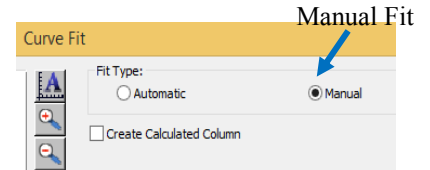
- The lower graph is rebound height vs. bounce number (that is, the data you just entered). Our model is an exponential function:  $y = ab^x$ , where  $y$  is the rebound height and  $x$  the bounce number.  $a$  and  $b$  are two parameters that you need to determine.  $a$  represents the starting height (sometimes called the zeroth bounce!), or the height before the first bounce. That is the height on the  $x = 0$  line of your data table, which is also the  $y$ -intercept.

**(S/B/CU/DO/S) Analysis Question 1.**

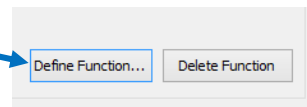
Use the function  $y = ab^x$  to explain why  $a$  is equal to the y-intercept.

4. To determine the parameter,  $b$ , you can plot the model equation and try different values for  $b$  until one fits best. First, you need to enter the exponential model.

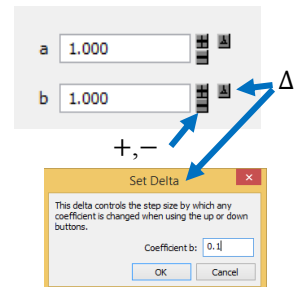
- a. (CU/S) Click once on the lower graph to make it active.
- b. (CU/S) From the Analyze menu (in top toolbar), choose Curve Fit. Select the Manual Fit Type option.



- c. (CU/S) Click the Define Function button.
- d. (CU/S) Enter the expression  $a * b^N$  in the Define Custom Function field. Click .



- e. (CU/S) Enter your ball's initial height in the  $a$  field.
- f. (CU/S) Note that the default value for  $b$  is 1.00. Click on the "+" and "-" buttons next to the  $b$  field to adjust the value until the model fits the data as best it can. To get an even more accurate model, click  $\Delta$  to make the step size smaller.
- g. (CU/S) Click  to return to the main graph.



**(S/CU/B/DO) Analysis Question 2.**

Use the parameters from Step 4 and write the optimized model equation.

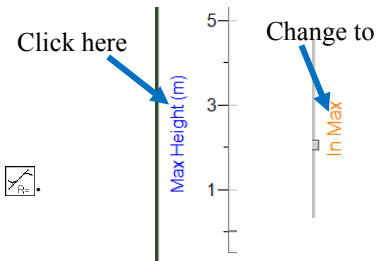
- h. (CU/S) Remove the fit by clicking the close box of the fit floating box on the graph.


5. The exponential equation  $y = ab^x$  can be made into a linear equation by taking the log of both sides:

$$\begin{aligned}
 y &= ab^x \\
 \ln y &= \ln(ab^x) \\
 &= \ln a + \ln(b^x) \\
 &= \ln a + x \ln b \\
 &= (\ln b)x + \ln a
 \end{aligned}$$

That is, a graph of  $\ln y$  vs.  $x$  is linear, with a slope of  $\ln b$  and y-intercept of  $\ln a$ .  
 Logger Pro can draw such a graph for you.

- (CU/S) Click the y-axis label on the lower graph.
- (CU/S) Choose  $\ln$  Max.



6. (CU/S) Since the graph is nearly linear, click the Linear Fit button, .

**(S/CU/B/DO) Analysis Question 3.**

Use the parameters on the screen to write the fitted linear equation.

**(S/CU/B/DO) Analysis Question 4.**

How well does the linear regression fit the data of  $\ln(\text{rebound height})$  versus bounce number?

**(S/CU/B/DO) Analysis Question 5.**

Use the values of  $a$  and  $b$  that you determined to find  $\ln a$  and  $\ln b$ . Record these values.

How do these values compare with the linear regression constants  $a$  and  $b$ ? Do they match?

Explain why they *should* match.

**(Hint:** Compare the equations  $\ln y = (\ln b)x + \ln a$  and  $y = mx + k$ .)

**(S/CU/B/DO) Analysis Question 6.**

Suppose that you repeat this activity using a ball that is not as resilient as the ball you used. For example, if you used a basketball, imagine that some air is let out of it, and then the exercise is repeated. Would the constants  $a$  and  $b$  in the equation  $y = ab^x$  be affected by this? How?