PreCal Review for Unit 6 Name:______________________________

Non-Calculator

Simplify:

1) $\sin\alpha\tan\alpha\sec\alpha\csc\alpha$

$\sin\alpha \cdot \frac{\sin\alpha}{\cos\alpha} \cdot \frac{1}{\cos\alpha} \cdot \frac{1}{\sin\alpha}$

$\frac{\sin\alpha}{\cos\alpha} \cdot \frac{1}{\cos\alpha} = \boxed{\tan\alpha\sec\alpha}$ or $\boxed{\frac{\sin\alpha}{\cos^2\alpha}}$

2) $\frac{(\cot\theta)^2}{1-(\sin\theta)^2} = \frac{\cot^2\theta}{1-\sin^2\theta} = \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\cos^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{1}{\cos^2\theta}$

$= \frac{1}{\sin^2\theta} = \boxed{\csc^2\theta}$

3) $\frac{\sin 2\beta}{\cos 2\beta - \cos^2\beta} = \frac{2\sin\beta\cos\beta}{\cos^2\beta - \sin^2\beta - \cos^2\beta}$

$= \frac{2\sin\beta\cos\beta}{-\sin^2\beta}$

$= \frac{-2\cos\beta}{\sin\beta} = \boxed{-2\cot\beta}$

4) $\frac{2}{1-\csc\gamma} - \frac{2}{1+\csc\gamma}$

$= \frac{2}{1-\csc\gamma} \cdot \frac{(1+\csc\gamma)}{(1+\csc\gamma)} - \frac{2}{1+\csc\gamma} \cdot \frac{(1+\csc\gamma)}{(1-\csc\gamma)}$

$= \frac{2+2\csc\gamma - 2 + 2\csc\gamma}{1-\csc^2\gamma}$

$= \frac{4\csc\gamma}{-\cot^2\gamma}$

$= \frac{\frac{4}{\sin\gamma}}{\frac{-\cos^2\gamma}{\sin^2\gamma}}$

$= \frac{4}{\sin\gamma} \cdot \frac{-\sin^2\gamma}{\cos^2\gamma}$

$= \boxed{\frac{-4\sin\gamma}{\cos^2\gamma}}$ or $\boxed{-4\tan\gamma\sec\gamma}$

5) $1 - 4\sin^2\theta\cos^2\theta$

$= 1 - (2\sin\theta\cos\theta)^2$

$= 1 - (\sin 2\theta)^2$

$= 1 - \sin^2 2\theta \rightarrow \boxed{\cos^2(2\theta)}$

6) $2\sin\alpha\cos^3\alpha + 2\sin^3\alpha\cos\alpha$

$= 2\sin\alpha\cos\alpha(\cos^2\alpha + \sin^2\alpha)$

$= 2\sin\alpha\cos\alpha(1)$

$= 2\sin\alpha\cos\alpha$

$\boxed{\sin 2\alpha}$

Prove the Identity:

7) $\tan^2 x - \sin^2 x = \sin^2 x\tan^2 x$

$\frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x(\cos^2 x)}{\cos^2 x} =$

$\frac{\sin^2 x - \sin^2 x\cos^2 x}{\cos^2 x} = \frac{\sin^2 x(1-\cos^2 x)}{\cos^2 x}$

$= \frac{\sin^2 x(\sin^2 x)}{\cos^2 x} = \tan^2 x\sin^2 x$ ✓

8) $\frac{\cos\sigma}{1-\tan\sigma} + \frac{\sin\sigma}{1-\cot\sigma} = \cos\sigma + \sin\sigma$ ✓

$\frac{\cos\sigma}{1-\frac{\sin\sigma}{\cos\sigma}} \cdot \frac{\cos\sigma}{\cos\sigma} + \frac{\sin\sigma}{1-\frac{\cos\sigma}{\sin\sigma}} \cdot \frac{\sin\sigma}{\sin\sigma} =$

$\frac{\cos^2\sigma}{\cos\sigma - \sin\sigma} + \frac{\sin^2\sigma}{\sin\sigma - \cos\sigma}$

$= \frac{\cos^2\sigma - \sin^2\sigma}{\cos\sigma - \sin\sigma}$

$= \frac{(\cos\sigma - \sin\sigma)(\cos\sigma + \sin\sigma)}{\cos\sigma - \sin\sigma}$

$= \cos\sigma + \sin\sigma$

9) $\sec x - \sin x\tan x = \cos x$

$\frac{1}{\cos x} - \sin x \cdot \frac{\sin x}{\cos x} =$

$\frac{1-\sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x}$

$= \cos x$ ✓

10) $\cos^2 2\theta - \cos^2\theta = \sin^2\theta - \sin^2 2\theta$ ✓

$= \sin^2\theta - (2\sin\theta\cos\theta)^2$

$= \sin^2\theta - 4\sin^2\theta\cos^2\theta$

$= \sin^2\theta(1 - 4\cos^2\theta)$

$= (1-\cos^2\theta)(1-4\cos^2\theta)$

$= 1 - 4\cos^2\theta - \cos^2\theta + 4\cos^4\theta$

$= 4\cos^4\theta - 4\cos^2\theta + 1 - \cos^2\theta$

$= (2\cos^2\theta - 1)(2\cos^2\theta - 1) - \cos^2\theta$

$= (\cos 2\theta)(\cos 2\theta) - \cos^2\theta$

$= \cos^2 2\theta - \cos^2\theta$

Solve on the interval $[0,2\pi)$:

11) $\cos 2x = \cos x$

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}, \cos x = 1$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0}$

12) $\sqrt{2}\sec x\sin x = \sec x$

$\sqrt{2}\sec x\sin x - \sec x = 0$

$\sec x(\sqrt{2}\sin x - 1) = 0$

$\sec x = 0 \qquad \sqrt{2}\sin x - 1 = 0$

$\frac{1}{\cos x} = 0$ undefined $\qquad \sin x = \frac{1}{\sqrt{2}}$

$\sin x = \frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}}$

13) $3\tan^2\theta = 1$

$\tan\theta = \pm\sqrt{\frac{1}{3}}$ or $\pm\frac{\sqrt{3}}{3}$

$\boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

Find the exact value of x:

14) $\sin\frac{5\pi}{12} = x$

$x = \sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

15) $\cos\frac{11\pi}{12} = x$

$\cos\frac{11\pi}{12} = \cos\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right)$

$= \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$= \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$ OR $\boxed{-\frac{\sqrt{2}+\sqrt{6}}{4}}$

Calculator:

Solve on the interval [0,2π). Round to the nearest thousandths.

16) $sin^2x + 0.5 = 3\cos x$

$x = 1.119$
$x = 5.164$

graph each side and get intersection

17) $x^2 = 10 - sin^4x$

$x = 3.162$

Prove the Identity algebraically and graphically.

→ graph left side in f1(x), graph right side in f2(x) } see that they are the same function (graph)

18) $\sin 4\theta = 2\sin 2\theta \cos 2\theta$

$\sin 2(2\theta) =$

$2\sin 2\theta \cos 2\theta =$ ✓

and graphically the left + right side are same graph

19) $\csc x + \cot x = \dfrac{\sin x}{1+\cos x}$

$\frac{1}{\sin x} + \frac{\cos x}{\sin x}$

$\frac{1+\cos x}{\sin x}$

$\frac{(1+\cos x)}{1+\cos x} \cdot \frac{1+\cos x}{\sin x}$

$\frac{1+2\cos x+\cos^2 x}{(1+\cos x)(\sin x)}$

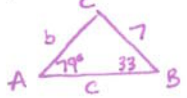
:(... algebraic proof is failing

not an identity, see graphically that $\csc x + \cot x$ is not same graph as $\frac{\sin x}{1+\cos x}$

In ΔABC Round to the nearest tenth.

20) Find given m∡A=79°, m∡B=33°, a=7

AAS — Law of Sines



$\frac{\sin 79°}{7} = \frac{\sin 33°}{b}$

$b\sin 79° = 7\sin 33°$

$b = \frac{7\sin 33°}{\sin 79°}$ → $b = 3.884$

$\angle C = 180 - (79° + 33°)$

$\angle C = 68°$

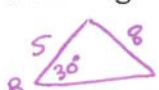
$\frac{\sin 79°}{7} = \frac{\sin 68°}{c}$

$c\sin 79° = 7\sin 68°$

$c = \frac{7\sin 68°}{\sin 79°}$ → $c = 6.612$

21) Find given a=5, b=8, m∡B =30°

SSA — how many Δs?



$\sin 30° = \frac{h}{5}$

$2.868 = h$

$8 > 2.868$ 2 Δs?

$\frac{\sin 30°}{8} = \frac{\sin B}{5}$

$8\sin B = 5\sin 30°$

$\sin B = \frac{5\sin 30}{8}$

$\angle B = \sin^{-1}\left(\frac{5\sin 30}{8}\right) = 18.210°$

$\angle C = 180 - (30° + \angle B)$

$\angle C = 131.790°$

$\frac{\sin 30°}{8} = \frac{\sin 131.790°}{c}$ → $c = \frac{8\sin 131.790°}{\sin 30°}$

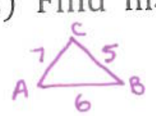
$c = 11.929$

$\angle B' = 180 - 18.210°$

$\angle B' = 161.790°$ → w/ $\angle A = 30°$ can't have 2nd Δ, so only one Δ

22) Find m∡A given a=5, b=7, c=6

SSS — Law of Cosines



$a^2 = b^2 + c^2 - 2bc\cos A$

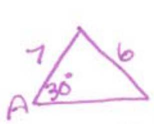
$5^2 = 7^2 + 6^2 - 2(7)(6)\cos A$

$\frac{5^2 - 7^2 - 6^2}{-2(7)(6)} = \cos A$ → $A = \cos^{-1}\left(\frac{5^2-7^2-6^2}{-2(7)(6)}\right)$

$\angle A = 44.415°$

23) Solve ΔABC given a=6, b=7, m∡A=30°

SSA — How many Δs?



$\sin 30° = \frac{h}{7}$

$3.5 = h$

$6 > 3.5$ 2 Δs?

$\frac{\sin 30°}{6} = \frac{\sin B}{7}$

$b\sin B = 7\sin 30°$

$\sin B = \frac{7\sin 30°}{6}$

$B = \sin^{-1}\left(\frac{7\sin 30°}{6}\right)$

$\angle B = 35.685°$

$\angle C = 180 - (\angle A + \angle B)$

$\angle C = 114.315°$

$\frac{\sin 30°}{6} = \frac{\sin 114.315°}{c}$

$c = \frac{6\sin 114.315°}{\sin 30°}$ → $c = 10.936$

2nd Δ

$\angle B' = 180 - \angle B$

$\angle B' = 144.315°$

$\angle C' = 180 - (\angle A + \angle B')$

$\angle C' = 5.685°$

$\frac{\sin 30°}{6} = \frac{\sin 5.685°}{c'}$

$c' = \frac{6\sin 5.685°}{\sin 30°}$

$c' = 1.189$