PreCal Review for Unit 7 NAME______________________________

Non-Calculator

1) *Find the sum of the coefficients of* $(4x-5y)^3$

$(4-5)^3 = (-1)^3 = \boxed{-1}$

2) *Find the sum of the first 328 even natural numbers.*

$n = 328$ $\quad a_{328} = 2 + (328-1)(2)$ $\quad S_{328} = \frac{328}{2}(2+656)$

$a_1 = 2$ $\quad = 2 + (327)(2)$ $\quad = \boxed{107912}$

$= 656$

3) *Find the 10th term of the geometric sequence if* $a_3 = \frac{1}{3}$ *and* $a_7 = 27$.

$a_3 = a_1 r^{3-1}$ $\quad a_7 = a_1 r^{7-1}$ $\quad 27 = a_1 r^2 \cdot r^4$ $\quad a_{10} = a_1 r^9$

$\frac{1}{3} = a_1 r^2$ $\quad 27 = a_1 r^6$ $\quad 27 = \frac{1}{3} r^4$ $\quad = a_1 r^2 \cdot r^7$

$81 = r^4 \rightarrow r = \pm 3$ $\quad = \frac{1}{3}(\pm 3)^7 = \boxed{\pm 729}$

4) *Find the sum of the infinite geometric series:* $10 + 4 + \frac{8}{5} + \frac{16}{25} + \cdots$

$S_\infty \Rightarrow \sum_{k=1}^{\infty} 10\left(\frac{2}{5}\right)^{k-1} = \frac{10}{1-2/5} = \frac{10}{3/5} = 10 \cdot \frac{5}{3} = \boxed{\frac{50}{3}}$

$r = 2/5$ $\quad \frac{4}{10} = \frac{2}{5}$ ... $\quad \frac{8/5}{4} = \frac{8}{5} \cdot \frac{1}{4} = \frac{2}{5}$

5) *Find the* n^{th} *term of the geometric sequence if:* $a_4 = 1$ *and* $a_8 = 81$.

$a_4 = a_1 r^3$ $\quad a_8 = a_1 r^7$ $\quad 1 = a_1 r^3$ $\quad a_n = a_1 r^{n-1}$

$1 = a_1 r^3$ $\quad 81 = a_1 r^3 \cdot r^4$ $\quad 1 = a_1 (\pm 3)^3$ $\quad = \boxed{\pm \frac{1}{27}(\pm 3)^{n-1}}$ OR $\pm \frac{1}{27} \cdot 3^n \cdot 3^{-1}$

$81 = 1 r^4 \longrightarrow r = \pm 3$ $\quad 1 = \pm 27 a_1$ $\quad = \pm \frac{1}{27} \cdot 3^n \cdot \frac{1}{3}$

$\pm \frac{1}{27} = a_1$ $\quad = \boxed{\pm \frac{1}{81} \cdot 3^n}$

6) *Find the summation:* $\sum_{n=1}^{6} -3\left(\frac{1}{2}\right)^{n-1}$

$\frac{-3(1-(\cdot 1/2)^6)}{1-1/2} = \frac{-3(1-\frac{1}{64})}{1/2} = \frac{-3(\frac{64}{64}-\frac{1}{64})}{1/2} = \frac{-3(\frac{63}{64})}{1/2} = -3\left(\frac{63}{64}\right) \cdot \frac{2}{1} = \boxed{\frac{-189}{32}}$

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7) *Find* a_n *for the arithmetic sequence with* $a_2 = -5,\ d = 4,\ \&\ n = 47$

$a_n = a_1 + (n-1)d$ $\quad a_{47} = a_1 + (47-1)d$

$a_2 = a_1 + (2-1)4$ $\quad = -9 + (46)(4)$

$-5 = a_1 + 4 \rightarrow a_1 = -9$ $\quad = \boxed{175}$

8) *Find the fifth term of* $(5-x)^{7 \leftarrow n}$

$r = 5$

$r - 1 = 4$

$\binom{7}{4}(5)^3(-x)^4 = \frac{7!}{4!(7-4)!} \cdot 125x^4 = \frac{7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4!}}{\cancel{4!}\ \cancel{3 \cdot 2} \cdot 1} \cdot 125x^4 = 35 \cdot 125x^4 = \boxed{4375x^4}$

9) *Find* $f(4)$ *if* $f(x) = \frac{(x+2)!}{(x)!}$ *by 2 different methods.*

$f(4) = \frac{(4+2)!}{4!} = \frac{6!}{4!} = \frac{6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = \boxed{30}$ } Method 1

$f(x) = \frac{(x+2)(x+1)\cancel{x!}}{\cancel{x!}} = (x+2)(x+1)$

$f(4) = (4+2)(4+1)$

$= 6 \cdot 5$

$= \boxed{30}$ } Method 2

10) *Find the summation:* $\displaystyle\sum_{n=1}^{9,999} \log \frac{n}{n+1}$

$\sum_{n=1}^{9999} (\log n - \log(n+1)) = (\log 1 - \log 2) + (\log 2 - \log 3) + (\log 3 - \log 4) +$

$\ldots (\log 9998 - \log 9999) + (\log 9999 - \log 10000)$

$= \log 1 - \cancel{\log 2} + \cancel{\log 2} - \cancel{\log 3} + \cancel{\log 3} - \cancel{\log 4} + \ldots + \cancel{\log 9998} - \cancel{\log 9999} + \cancel{\log 9999} - \log 10000$

$= \log 1 - \log 10000 \Rightarrow -\log 10000 \Rightarrow -\log 10^4 \Rightarrow \boxed{-4}$

Calculator

11) $Find\ the\ partial\ sum\ of\ \sum_{x=1}^{79} \log_{\pi} x$ $= \boxed{235.244}$

12) $What\ is\ the\ 12th\ term\ of\ (1.5x - 2.1y)^{14}$

$r = 12$

$r-1 = 11$

$\binom{14}{11}(1.5x)^3(-2.1y)^{11}$

$= \boxed{-4.303 \times 10^6 \cdot x^3 y^{11}}$

13) $Find\ the\ formula\ for\ a_n\ and\ find\ a_1\ for\ the\ arithmetic\ sequence:$

$a_4 = -23, \quad a_8 = 95$

$a_4 = a_1 + (4-1)d$

$-23 = a_1 + 3d$

$a_8 = a_1 + (8-1)d$

$95 = a_1 + 7d$

$-23 = a_1 + 3d$

$-(95 = a_1 + 7d)$

$-118 = -4d$

$29.5 = d$

$-23 = a_1 + 3(29.5)$

$-23 = a_1 + 88.5$

$\boxed{-111.5 = a_1}$

$a_n = a_1 + (n-1)d$

$= -111.5 + (n-1)29.5$

$= -111.5 + 29.5n - 29.5$

$\boxed{a_n = 29.5n - 141}$

14) $Find\ the\ summation\ by\ 2\ methods:\ \sum_{24}^{95} 1.6\left(\frac{2}{3}\right)^x$ $= \boxed{2.851 \times 10^{-4}}$ } Method 1

OR

$\frac{a_1(1-r^n)}{1-r} = \frac{9.505 \times 10^{-5}(1-(2/3)^{72})}{1-2/3} = 2.851 \times 10^{-4}$ } Method 2

15) $Find\ the\ formula\ for\ a_n\ and\ find\ a_1\ for\ the\ geometric\ sequence:$

$a_3 = \frac{25}{7}\ and\ a_7 = \frac{15625}{16807}$

$\frac{25}{7} = a_1 r^2$

$\frac{15625}{16807} = a_1 r^6$

$\frac{15625}{16807} = a_1 r^2 \cdot r^4$

$\frac{15625}{16807} = \frac{25}{7} r^4$

$\frac{625}{2401} = r^4$

$\pm \frac{5}{7} = r$

$\frac{25}{7} = a_1 (\pm \frac{5}{7})^2$

$\frac{25}{7} = a_1 (\frac{25}{49})$

$\boxed{7 = a_1}$

$a_n = a_1 r^{n-1}$

$\boxed{a_n = 7(\pm \frac{5}{7})^{n-1}}$